Tribhuvan University
Institute Of Science and Technology
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| Course Title: Linear Programming | Full Marks: 50 |
| Course No. : Math 403 | Pass Mark: 17.5 |
| Level : B.Sc. | Year: IV |
| Nature of Course: Theory & problem solving | Period per week: 5 |

Teaching hour: 3 teaching + 2 problem solving periods per week

**Course Objective**

This course aims at introducing techniques of and the solution status of the related problems in linear programming (LP).

After the completion of this study, students will be familiar with LP models and their real life applications.

Students will be able to tackle the LP solution algorithms.

**Detailed Course**:

**Unit 1 Mathematical Background** [9 + 6 teaching hrs]

system of linear equations and inequalities,

basic solutions,

Degenerate and non-degenerate

Necessary and sufficient condition for the existence of non-degeneracy

i.e. A basic feasible solution is non-degenerate if and only if the corresponding column vectors are linearly independent.

lines and hyperplanes,

convex sets and functions,

convex sets and hyperplanes,

convex cones,

polyhedron,

A hyperplane is a convex set.

A closed half space is a convex set.

The set of all convex combinations of a finite number of points is a convex set.

The set of all feasible solutions of an LP problem is a convex set.

The intersection of any number of convex sets is also convex.

concepts on graphs,

time complexity and reduction relations,

related exercise.

**Unit 2 LP-Models and Complexity** [9 + 6 teaching hrs]

variables and constraints,

cost function,

two-variable LP models

graphical solution methods,

general, canonical and standard forms of LP models,

slack and surplus variables,

the equivalency of different LP-forms,

decision versions of the optimization problems,

related exercise.

**Unit 3 Simplex Algorithm** [9 + 6 teaching hrs]

extreme points,

basic feasible solutions,

Reduction of any feasible solution into a basic feasible solution,

Improvement of basic feasible solution,

If an LP problem has at least one feasible solution, then it has at least one basic feasible solution,

If an LP problem has an optimal solution, then at least one feasible solution must be the optimal one,

Unbounded solutions,

Optimality conditions,

Extreme points and basic feasible solutions, solution of LP problems by simplex method,

complexity of simplex method,

relation of extreme points and basic feasible solutions,

Simplex Algorithm,

Selection of the vector to enter the basis,

Degeneracy and breaking ties,

Transformation formula,

The initial basic feasible solution-artificial variables,

Inconsistency and redundancy,

Tableau format and its use,

Conversion of a minimization problem to a maximization problem,

Two phase method,

related exercise.

**Unit 4 LP Duality Theory** [9 + 6 teaching hrs]

Alternate formulations of LPs,

the dual LP formulation,

Fundamental properties of dual problems,

Other formulations of dual problems,

complementary slackness conditions,

Unbounded solution in the primal,

the dual simplex algorithm,

related exercise.

**Unit 5 Applications** [9 + 6 teaching hrs]

LP formulations of a given instance of

Models of transportation problem,

assignment problem,

maximal flows in a network,

minimum cost flow problem,

transshipment problem,

diet problem,

LP models of scheduling problems,

production planning,

scheduling problems,

related exercise.

**Text/ Reference books**

1. G. Hadley; Linear Programming, Narosa, Publishing House.
2. Hamdy A. Taha; Operations Research, Prentice Hall-Pearson.
3. Lueberger, D.G; Linear and Nonlinear Programming, Addison-Wesley.

**Guidelines to the question setters**

There will be 5 questions each carrying 10 marks and there will be a head, a tail and middle part if possible. All the questions are compulsory. There will be **two** OR choices in any question number from the same unit. The examination period of Math 403 will be 2 hours.

On the basis of the guidelines mentioned, we enclose one set of model question for Linear Programming (Math 403)

MODEL QUESTION

# Tribhuvan University

Bachelor Level / IV year/ Sc. & Tech. Full Marks: 50

Linear Programming (Math 403) Time: 2 Hours

*Candidates are required to give their answers in their own words as far as practicable.*

*The figures in the margin indicate full marks.*

**Attempt All Questions.**

1.(a)Define a hyper plane and a convex set. Show that a hyperplane is a convex set. [1+1+3]

(b) Prove that the intersection of two convex sets is also convex. [5]

2. (a) Write an example of a general form of a linear programming (LP) problem. Transform the example into the canonical and the standard forms. [2+3]

(b) Prove that the three forms of an LP problem are equivalent. [5]

3. (a) What do you mean by a basic feasible solution (bfs)? When is this called to be a degenerate bfs ? Describe the relationship between the extreme points and the bfs. [1+1+2]

(b) An optimal solution of an LP problem can be obtained by using the simplex method. Describe the method. [6]

**OR**

Solve the following LP problem with the Simplex approach.

$$maximize\ z = 7x_1 + 10x_2$$

s.t.

$$-x_1 + 3x_2 \leq 6$$

$$7x_1 + x_2 \leq 35$$

$$x_1, x_2 \geq 0$$ [10]

4. (a) What are the primal and the dual LP problems ? Write an example of a primal LP problem and then convert it into the dual form. [2+3]

(b) i. If $i^{th}$ variable appears in an optimal basic solution to the primal LP problem, then prove that the $i^{th}$ dual constraint holds a strict equality in the corresponding optimal solution to the dual. [4]

ii. Why is the aforementioned statement called the complementary slackness? [1]

5. Consider a network $[N; A]$ with $N = \{s, x, y, z, u, v, t\}$ and
$A = \{(s,x), (s,y), (s,z), (x,u), (x,v), (x,t), (y,u), (y,v), (z,v), (u,v), (u,t), (v,t)\}$

(a) Give appropriate costs to each arc in the network. [2]

(b) Represent the network in a graph form. [3]

(c) Define the maximum network problem. Develop an LP model of the problem. [1+4]

OR

5(a) Define the transportation problem with m sources are to be transported in n destinations. [3]

(b) Model the transportation problem into a general LP model. [5]

(c) What is the necessary and sufficient condition for the problem to have a feasible solution? [2]