Tribhuvan University

Institute of Science and Technology

Central Department of Mathematics

Course of Study for Four Year Mathematics

Course Title: **Mathematical Analysis II** *Course No.*: **Math 302**
Level : **B.Sc./B.A.** *Year*: **III**
Nature of Course: **Theory** *Full Marks*: **75**
Period per week: **9 (6 Theory + 3 tutorial)** *Pass Marks*: **27**

Course description

Mathematical Analysis – II (Math 302) is a compulsory course to be taught in the third year. This course is designed to provide advanced knowledge of analysis to students offering mathematics as a major subject. Pre-requisite for this course is Mathematical Analysis I, which the students have studied in the second year. This course is divided into 10 units. The total number of lecture hours is expected to be 150 (1 lecture hour = 45 minutes) for the completion of this course.

Course objectives

The general objectives of this course are

a) To develop theoretical knowledge and analytical skill in the emerging areas of mathematics

b) To raise interest of students in the field of analytical world so that they can take up any course easily in modern mathematics.

c) To acquire and develop skill in the use and understanding of mathematical language.

d) To acquire knowledge an understanding of the language of mathematical terms, symbols, statements formaulae, definitions, logic etc.

e) To construct solutions and proofs with their own independent efforts.

f) To prepare a sound base for higher studies in Mathematics.

Teaching Method

As the nature of this course is theoretical emphasis should be given to analytical theory and consequently much time has been allotted for theory part. For the better understanding of the theory, examples, counter examples and related problems are very essential. So, while teaching, examples and related problems should be discussed along with the theorem wherever possible. Also, it would be better to encourage the students to solve the problems as much as possible on their own.

Lesson Plan

Lesson plan is necessary for every teacher. But it is very difficult to prepare it. Preparation of lesson plan is affected by various factors such as time duration, number of students in the class room, ability and seriousness of the students and , of course, teachers themselves. Even for an experienced teacher. Preparation of lesson plan is a matter of challenge as it depends on various factors mentioned above. In this paper, we have roughly estimated the necessary lecture hours of 45/50 minutes for different units and sub-units to complete this course.

Evaluation

Evaluation is the integral part of teaching and learning process. In our system, examination is only a part of the process of evaluation. To evaluate the students in this course, questions should carry definitions, theorems, examples as well as problems related to the theorems. The questions should discourage guessing and bluffing. The use of 'catch' questions should be minimized. At the same time the questions should not aim at confusing or puzzling the students.

1. There will be **two Groups**: A and B. Group A will contain **5 questions** with **two OR-questions**, each carrying **7 marks**. Group B will contain **10 questions** with **four OR-questions**, each carrying **4 marks**.
2. Either **one question carrying 7** marks or **two questions, each carrying 4** marks will be asked from each unit.
3. OR-questions will be asked from the **same units**.
4. The students are required to **attempt all questions** from Group A and all questions from Group B.

<u>**Course Contents**</u>

Unit 1 Euclidean spaces and metric spaces: **12 Lecture hours**

Set $\measuredangle^n$, Algebraic structure of $\measuredangle^n$, Metric structure of $\measuredangle^n$, Cauchy-Schwarz Inequality, Topology in $\measuredangle^n$, Metric spaces, Pointset topology in metric spaces.

Unit 2 Compactness **8 Lecture hours**

Bolzano-Weiererstrass theorem, Cantor intersection theorem, Lindelof covering theorem, Heine-Borel covering theorem, Compactness in $\measuredangle^n$, Compactness of a metric space.

Unit 3 Sequences in metric spaces: 16 Lecture hours

Convergent sequence in a metric space. Cauchy sequences, Complete metric spaces, Contraction Mapping Theorem, Sequences and Compactness, Bolzano-Weiererstrass theorem for sequences, Nearest Points.

Unit 4 Limits and Continuity: 11 Lecture hours

Limits of a function, Continuous functions, Continuity of composite functions. Continuity and inverse images, Functions continuous on compact sets, Bolzano's theorem and intermediate value theorem, Uniform continuity, Uniform continuity and compact sets.

Unit 5 Multivariable Differentiation: 22 Lecture hours

Linear operator and its matrix representation, Total derivative, Partial Derivatives, Directional derivatives, Jacobean matrix, Chain rule and its matrix form, Mean Value theorem, Higher order partial derivatives, Sufficient condition for equality of mixed partial derivatives.

Unit 6 Functions of Bounded Variation: 9 Lecture hours

Properties of monotonic functions, functions of bounded variation, Total variation, Its additive property. Total variation on $[a, x]$ as a function of x, Functions of bounded variation expressed as the difference of increasing functions, Continuous function of bounded variation.

Unit 7 Riemann-Stieltjes Integration I: 17 Lecture hours

Riemann-Stieltjes integrals, Linear properties, Integration by parts, Change of variable, Reduction to a Riemann integral, Step-functions as integrators, Reduction to a finite sum, Increasing integrators, Upper and lower integrals, Riemann's condition, Comparison theorems, Integrators of bounded variation.

Unit 8 Riemann-Stieltjes Integration II: 11 Lecture hours

Necessary and sufficient conditions for existence of Riemann-Stieltjes integrals, Mean Value theorem, Integral as a function of the interval, Second Fundamental theorem, Second Mean Value theorem, Riemann integrals depending on a parameter, Differentiation under the integral sign, Interchanging the order of Riemann integrations.

Unit 9 Sequences and series of functions 19 Lecture hours

Sequences of Functions: Pointwise convergence, Uniform convergence, Criterion for non-uniform convergence, Cauchy Condition for Uniform Convergence, Uniform convergence and continuity, Uniform convergence and integration, Uniform convergence and differentiation, Dini's Theorem;

Series of functions: Uniform convergence of series of functions, Cauchy condition, Weierstrass M-test, Dirichlet's test, and Abel's test for uniform convergence. Uniform convergence and continuity, Uniform convergence and integration, Uniform convergence and differentiation.

Unit 10 Improper Integrals 18 Lecture hours

Classification of improper integrals, Convergence, Divergence, Application of Fundamental Theorem of calculus. Simple properties, Conditions and tests for convergence, Absolute convergence, Abel's test and dirichlet's test.

Reference books:

1. Brian S. Thomson, Judith B. Bruckner, Andrew M. Bruckner, *Elementary real analysis*
2. Tom Apostol, *Mathematical Analysis*, Narosa Publishing House, India.
3. R. G. Bartle, *The Elements of Real Analysis*, John Wiley and Sons.
4. S. Ponnusamy, *Foundations of Mathematical Analysis*, Springer.
5. V. A. Zorich, *Mathematical Analysis* I and II, Springer
6. David V. Widder, *Advanced Calculus*, Prentice Hall.
7. N. P. Pahari, A Textbook of Mathematical Analysis, Sukunda Pustak Bhawan