Name of the student:-

Roll No:-. . .

Tribhuvan University

Institute of Science and Technology

M.A. / M.Sc. Entrance Examination (Mathematics)

2079

Time:.2 Hours Full Marks: 100

Attempt 100 questions (from 1 to 90) and remaining 10 (from 91 to 100 either Mechanics or Linear Programming). 1×100

Tick (✓) the best alternatives.

1. What is the value of h for which the matrix $\left(\begin{array}{cc|c} 2 & h & 4 \\ 3 & 6 & 7 \end{array}\right)$ is augmented matrix of an inconsistent system?
   (a) 2
   (b) 3
   (c) 4
   (d) 0

2. Which is of these transformations is linear?
   (a) $T(x, y) = (3x^2, 4y)$
   (b) $T(x, y) = (xy, x + y)$
   (c) $T(x, y) = (0, 0)$
   (d) $T(x, y) = (x - y, x/y)$

3. What is the numerical value of the product $AB = \begin{pmatrix} 1 & 3 & 2 & -5 \\ 2 & 2 & -3 & 4 \\ 5 & 1 & 1 & 6 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ -5 & 0 \\ 4 & 1 \end{pmatrix}$?
   (a) 10
   (b) 3
   (c) 16
   (d) not defined.

4. Let $v_1 = (1, 3, -3), v_2 = (3, 10, -1), v_3 = (-2, -1, h)$. For what value of h is $\{v_1, v_2, v_3\}$ linearly dependent?
   (a) 0
   (b) 46
   (c) 6
   (d) 64

5. Let $T(x, y, z) = (3x - 2y, 5y + z, z - x)$. What is the standard matrix for T?
   (a) $\begin{pmatrix} 3 & 0 & -1 \\ -2 & 5 & 0 \\ 0 & 1 & 1 \end{pmatrix}$
   (b) $\begin{pmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
   (c) $\begin{pmatrix} 3 & -2 & 0 \\ 0 & 5 & 1 \\ -1 & 0 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

6. Let $B = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 7 & 8 \\ 1 & 0 & 6 \end{pmatrix}$ and $A = B^{-1}$. What is a_{21}?

   (a) -9
   (b) 3
   (c) -10
   (d) 6

7. What is the determinant of $\begin{vmatrix} 0 & 1 & -1 & 1 \\ 1 & 7 & 9 & 11 \\ 0 & 4 & 2 & 1 \\ 0 & 1 & 2 & 3 \end{vmatrix}$

   (a) 21
   (b) -6
   (c) 6
   (d) -21

8. Which set spans $\mathbb{R}^3$?

   (a) span $\left\{ \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\}$
   (b) span $\left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$
   (c) span $\left\{ \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} \right\}$
   (d) span $\left\{ \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}$

9. The eigenvalue of $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ are

   (a) 3, 1
   (b) -5, 1
   (c) 5, -1
   (d) 6, -2

10. Consider the two vectors $u = (-2, -3)$ and $v = (-3, 4)$. What is $|| < u, v > v||$?

   (a) 30
   (b) -6
   (c) 5
   (d) 10

11. Which set forms a group?

   (a) $(\mathbb{Z}, .)$
   (b) $(\mathbb{Z}, +)$
   (c) $(\mathbb{Q}^+, +)$
   (d) $(\mathbb{Z}^+, +)$

12. Let $S = \mathbb{R} - \{-1\}$.Define $*$ on S by $a * b = a + b + ab$. Then the solution of $3 * x * 4 = -21$ in S is

   (a) $-3/2$
   (b) -2
   (c) 2
   (d) 0

13. What is the value of $(pq)^{-1}$ if $p = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}, q = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$?

   (a) (1 2)
   (b) (1)
   (c) (1 3)
   (d) (2 3)

14. Determine whether the given function is a permutation of $\mathbb{R}$.

   (a) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$
   (b) $g : \mathbb{R} \to \mathbb{R}$ defined by $g(x) = e^x$
   (c) $h : \mathbb{R} \to \mathbb{R}$ defined by $h(x) = x + 1$
   (d) $k : \mathbb{R} \to \mathbb{R}$ defined by $k(x) = Lnx$

15. Let $\phi : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ where $\phi(1, 0) = 3$ and $\phi(0, 1) = -5$. What is the value of $\phi(-3, 2)$?

   (a) 24
   (b) -19
   (c) 19
   (d) 24.

16. What is the order of $\dfrac{\mathbb{Z}_6}{< 3 >}$?

   (a) 3
   (b) 4
   (c) 36
   (d) 6.

17. What is the degree for the field extension $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ over $\mathbb{Q}$?

   (a) 2
   (b) 3
   (c) 4
   (d) 0.

18. Compute the product in the given ring $(-3, 5)(2, -4)$ in $\mathbb{Z}_4 \times \mathbb{Z}_{11}$.

   (a) $(1, 1)$
   (b) $(2, 7)$
   (c) $(2, 5)$
   (d) $(2, 2)$.

19. Which is the maximal ideal of $\mathbb{Z}$?

   (a) $4\mathbb{Z}$
   (b) $8\mathbb{Z}$
   (c) $2\mathbb{Z}$
   (d) $\mathbb{Z}$.

20. The solution of the equation $x^3 - 7x^2 + 36 = 0$ when one root is double of another

(a) 1, 2, 4
(b) 6, 3, 1
(c) 3, 6, −2
(d) 1, 3, −6

21. If p and q are two statements such that p is true and q is false then which of the following statement is not true?

(a) $p \vee q$
(b) $p \wedge (\sim q)$
(c) $p \Rightarrow q$
(d) $q \Rightarrow p$

22. If A is any set and ϕ is empty set then which of the following is not true ?

(a) $\phi \subseteq \phi$
(b) $\phi \in \{\phi\}$
(c) $\phi = \{0\}$
(d) $\phi \subseteq A$

23. The domain of the function $f(x) = \sqrt{x-5}$ is

(a) $[5, \infty)$
(b) $(0, 5)$
(c) $(5, \infty)$
(d) $(0, 5]$

24. Let $S = \left\{ x : x = \frac{3n+2}{n}, \text{ where n is a positive integer} \right\}$ then greatest lower bound of S is equal to

(a) 10
(b) 2/3
(c) 5
(d) 3

25. Let $F = \left\{ A_n = (-\frac{1}{n}, \frac{1}{n}), n \in \mathbb{Z}^+ \right\}$. Then $\cap_{n \in \mathbb{Z}^+} A_n$ is equal to

(a) ϕ
(b) $\{0\}$
(c) $\{-1, 1\}$
(d) $(-1, 1)$

26. The set $S = \{1, -1, 1/2, -1/2, 1/3, -1/3, ...\}$ is

(a) open
(b) closed
(c) both open and closed
(d) neither open nor closed.

27. The sequence $\{(-n)^n\}$ is

(a) bounded above
(b) bounded below
(c) bounded above and below
(d) unbounded

28. Every Cauchy sequence in $\mathbb{R}$

(a) diverges
(b) converges
(c) is not bounded
(d) may not necessarily converge

29. An infinite series $\sum a_n$ is called absolutely convergent if
   (a) $|\sum a_n|$ is convergent
   (b) $|\sum a_n|$ is convergent but $\sum a_n$ is divergent
   (c) $\sum |a_n|$ is convergent
   (d) $\sum a_n$ is divergent

30. If the sequence is monotonically increasing and bounded above then it
   (a) converges to its infimum
   (b) converges to supremum
   (c) may or may not converge
   (d) diverges

31. If $f : \mathbb{R} \to \mathbb{R}$ is defined $f(x) = |x|$ for all $x \in \mathbb{R}$ then which of the following is true?
   (a) f is differential at $x = 0$
   (b) f is discontinuous at $x = 0$
   (c) f is continuous as well as differential at that point
   (d) f is continuous at $x = 0$ but not differential at that point.

32. Let $P = \{0, 1/3, 1/2, 3/4, 1\}$ be a partition of the closed interval [0,1]. The norm of P is
   (a) 1/3
   (b) 1/4
   (c) 1/6
   (d) 1

33. Let S be a subset of a metric space M; $\bar{S}$ be the closure of S and $S^{'}$ be the derived set of S. The relation between S, $\bar{S}$ and $S^{'}$ is
   (a) $S^{'} = S \cup \bar{S}$
   (b) $\bar{S} = S \cup S^{'}$
   (c) $S^{'} = S \cap \bar{S}$
   (d) $\bar{S} = S \cap S^{'}$

34. Which of the following set is not compact in $\mathbb{R}$?
   (a) $[0, 5]$
   (b) $[-1, 1] \cup [3, 4]$
   (c) $[0, 1]$
   (d) $[0, \infty)$

35. Which of the following is the fixed point of the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2 - 3x + 4$?
   (a) 0
   (b) 1
   (c) 2
   (d) 5

36. The integral $\int_1^\infty \frac{1}{x^p} dx$ is convergent if
   (a) $p = 1$

(b) $p < 1$

(c) $p \leq 1$

(d) $p > 1$

37. If $f(x,y) = x^3 + x^2y^3 - 2y^2$, then $f_y(2,1)$ is

(a) 8

(b) 16

(c) 32

(d) 64

38. The value of $\int_0^\pi x \; d(\sin x)$ is

(a) 2

(b) 0

(c) -1

(d) -2

39. Which of the following is not true?

(a) the set ϕ and $\mathbb{R}$ are open sets in $\mathbb{R}$

(b) the union of any number of open sets in $\mathbb{R}$ is open in $\mathbb{R}$

(c) the intersection of any number of open sets in $\mathbb{R}$ is open in $\mathbb{R}$

(d) the intersection of a finite number of open sets in $\mathbb{R}$ is open in $\mathbb{R}$

40. Suppose that f is a function that is bounded on an interval $I = [a,b]$ and α is monotonically increasing on I. Then $f \in \mathbb{R}(\alpha)$ on I if and only if for $\epsilon > 0$ there exists a partition P of I such that

(a) $U(P,f,\alpha) + L(P,f,\alpha) < \epsilon$

(b) $U(P,f,\alpha) + L(P,f,\alpha) > \epsilon$

(c) $U(P,f,\alpha) - L(P,f,\alpha) < \epsilon$

(d) $U(P,f,\alpha) - L(P,f,\alpha) > \epsilon$

41. In order to satisfy $f'(p) = 0$ for some $p \in (a,b)$, the following must hold

(a) f is continuous in the closed interval $[a,b]$

(b) f is differentiable in the open interval (a,b)

(c) $f(a) = f(b)$

(d) all of the above must hold

42. The equation $r = a(1 - \cos\theta)$ represents

(a) cycloid

(b) parabola

(c) cardioide

(d) ellipse

43. The correct value of $\lim_{x \to 0} \frac{1}{x} \log(1+x)$ is

(a) -1

(b) 1

(c) 0

(d) e

44. The correct statement is

(a) the sum of two continuous functions is a continuous function

(b) the difference of two continuous functions is a continuous function

(c) both statements (a) and (b) are correct

(d) both statements (a) and (b) are false

45. The value of $\lim_{x \to 0} \frac{\log \cos x}{\tan^2 x}$ is

(a) 1

(b) $\frac{1}{2}$

(c) $e^{-\frac{1}{2}}$

(d) $-\frac{1}{2}$

46. The L' Hospital's rule is the indeterminate form

(a) $0 \times \infty$

(b) $\frac{0}{0}$

(c) $\frac{\infty}{\infty}$

(d) all of the above

47. If $f(x, y)$ be a homogeneous function of x and y of degree n, then the Euler's theorem states that

(a) $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = f^n(x, y)$

(b) $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = f(x, y)$

(c) $x\frac{\partial f}{\partial x} - y\frac{\partial f}{\partial y} = nf(x, y)$

(d) $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf(x, y)$

48. The radius of curvature of the parabola $y^2 = 4ax$ at the vertex $(0, 0)$ is

(a) 0

(b) $2a$

(c) 2

(d) none of the above

49. If ϕ be the angle between the tangent and radius vector at any point on the curve $r = f(\theta)$, then the true statement is

(a) $\tan \phi = \frac{d\theta}{dr}$

(b) $\tan \phi = r\frac{d\theta}{dr}$

(c) $\cos \phi = \frac{d\theta}{ds}$

(d) $\sin \phi = r\frac{dr}{ds}$

50. Derivative of a constant real valued function is

(a) 0

(b) 1

(c) -1

(d) none of the above

51. The following statement is false

(a) each differential function is continuous at that point

(b) if a function is continuous at a point, then its limit exists at the point

(c) meaning of a continuous function is that both side limit and equals exist at that point

(d) every continuous function is differentiable

52. Choose the correct option

(a) the value of $\int \frac{dx}{x^2+a^2}$ is $\tan^{-1}\frac{x}{a}$, where $a \neq 0$

(b) the value of $\int \frac{dx}{x^2+a^2}$ is $\frac{1}{a}\tan^{-1}\frac{x}{a}$, where $a \neq 0$

(c) the value of $\int \frac{dx}{x^2-a^2}$ is $\frac{1}{a}\tan^{-1}\frac{x}{a}$, where $a \neq 0$

(d) none of the above

53. $\int \log x \, dx$ equals

(a) $x \log x + x$

(b) $\log x - x$

(c) $x \log x - x$

(d) $\log x + x$

54. The value of the definite integral $\int_0^1 \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$ is

(a) $\frac{1}{8}\pi^2$

(b) $\frac{1}{2}\pi^2$

(c) π^2

(d) $\frac{1}{8}\pi$

55. The correct value of the definite integral $\int_0^{\frac{\pi}{2}} \log \tan x \, dx$ is

(a) 0

(b) 1

(c) -1

(d) none of the above

56. The area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ between the major and minor axes is

(a) ab

(b) $\frac{1}{4}\pi ab$

(c) $\frac{1}{4}ab$

(d) πab

57. The integration $\int \cot x \, dx$ equals

(a) $\log \mid \sec x \mid$

(b) $\log \mid \sin x \mid$

(c) $\log \mid \tan x \mid$

(d) none of the above

58. To find the area between two curves $y = f(x)$ and $y = g(x)$ from a to b, the following step is not required

(a) graph the curves and draw a respective rectangle
(b) write a formula for $f(x) - g(x)$ in a simplified form if required
(c) differentiate the function $f(x) - g(x)$ at a point
(d) integrate the function $f(x) - g(x)$ within the limits

59. The area between the curves $y = \sec^2 x$ and $y = \sin x$ from 0 to $\frac{\pi}{4}$ is

(a) 1
(b) $\sqrt{2}$
(c) $\frac{1}{\sqrt{2}}$
(d) 0

60. The value of the integral $\int_{-a}^{+a} \frac{xe^{x^2}}{1+x^2}\, dx$ is

(a) 1
(b) -1
(c) π
(d) 0

61. The equation of ellipse with foci $(\pm 2, 0)$ and the eccentricity $1/2$ is

(a) $\frac{x^2}{16} + \frac{y^2}{12} = 1$
(b) $\frac{x^2}{16} + \frac{y^2}{25} = 1$
(c) $\frac{x^2}{9} + \frac{y^2}{16} = 1$
(d) none of the above

62. The point of contact of the line $y\sqrt{3} = x + 3$ and the ellipse $2x^2 + 3y^2 = 6$ is

(a) $(1, 2)$
(b) $(-1, \frac{2}{\sqrt{3}})$
(c) $(\frac{2}{\sqrt{3}}, 2)$
(d) $(-\frac{2}{\sqrt{3}}, 3)$

63. The equation of the hyperbola with focus $(2, 0)$, directrix $x - y = 0$ and eccentricity 2 is

(a) $x^2 - y^2 + 4xy4x + 5 = 0$
(b) $x^2 + y^2 - 4xy + 4x - 4 = 0$
(c) $x^2 + y^2 - 6xy + 4y - 4 = 0$
(d) none of the above

64. The angle between the lines joining the origin to the points of intersection of the line $y = 3x + 2$ with the curve $x^2 + 2xy + 3y^2 + 4x + 8y - 11 = 0$ is

(a) $\tan^{-1} \frac{2\sqrt{2}}{3}$
(b) $\tan^{-1} \frac{2\sqrt{2}}{5}$
(c) $\tan^{-1} \frac{5\sqrt{2}}{3}$
(d) None of these.

65. The conic section means

(a) parabola

(b) ellipse
(c) hyperbola
(d) all of the above

66. The equation of the common tangents of the parabolas $y^2 = 4ax$ and $x^2 = 4by$ is

(a) $b^{\frac{1}{3}}y + a^{\frac{1}{3}}x + a^{\frac{2}{3}}b^{\frac{2}{3}} = 0$
(b) $b^{\frac{1}{3}}y + a^{\frac{1}{3}}x + a^{\frac{1}{3}}b^{\frac{1}{3}} = 0$
(c) $b^{\frac{2}{3}}y + a^{\frac{2}{3}}x + a^{\frac{2}{3}}b^{\frac{2}{3}} = 0$
(d) $b^{\frac{1}{3}}y + a^{\frac{1}{3}}x + a^{\frac{5}{3}}b^{\frac{7}{3}} = 0$

67. The locus of the poles of normal chords of the parabola $y^2 = 4ax$ is

(a) $x^2 + y^2 = a^2$
(b) $y^2(2a + x) + 4a^3 = 5$
(c) $y^2(2a + x) + 4a^3 = 0$
(d) None of the above

68. The equation of the sphere passing through the points $(0, 0, 0), (0, 1, -1), (-1, 2, 0), \text{and}(1, 2, 3)$ is

(a) $7(x^2 + y^2 + z^2) - 15x - 25y - 11z = 0$
(b) $x^2 + y^2 + z^2 = 7\sqrt{3}$
(c) $x^2 + y^2 + z^2 + x + 2y + z = 7\sqrt{6}$
(d) $5x + 2y + 3z = 7\sqrt{5}$

69. The equation of the tangent plane at $(-1, 4, -2)$ of the sphere is

(a) $2x + 2y - 3z = 7$
(b) $2x - 2y + z + 12 = 0$
(c) $x + 2y - z = 8\sqrt{6}$
(d) $5x + 2y + 3z = 10$

70. The equation of the sphere for which the circle $2x + 3y + 4z = 8, x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$ being a great circle is

(a) $x^2 + y^2 + z^2 + 2x + 4y + 6z + 10 = 0$
(b) $x^2 + y^2 + z^2 - 2x + 4y - 6z + 11 = 0$
(c) $x^2 + y^2 + z^2 - 2x + 4y - 6z + 10 = 0$
(d) $x^2 + y^2 + z^2 - 8x - 5y - 6z + 15 = 0$

71. The equation of the plane of contact of a point $P(x_1, y_1, z_1)$ with respect to sphere $x^2 + y^2 + z^2 = r^2$ is

(a) $xx_1 - yy_1 - zz_1 = r^2$
(b) $xx_1 + yy_1 + zz_1 + r^2 = 0$
(c) $xx_1 + yy_1 + zz_1 = r^2$
(d) $xx_1 + yy_1 + zz_1 = 0$

72. The equation of the cone whose vertex is the orighin and base the circle $x = a, y^2 + z^2 = b^2$ is

(a) $b^2x^2 + a^2(y^2 + z^2) = 0$
(b) $xx_1 + yy_1 + zz_1 + r^2 = 0$
(c) $b^2x^2 - a^2(y^2 + z^2) = b^2$
(d) $b^2x^2 - a^2(y^2 + z^2) = 0$

73. The equations of the circular cones which contain the three co-ordinate axes are generators are

(a) $xy \pm yx \pm zx = 0$
(b) $3(x^2 + 2y^2z^2) + 2(4yz - zx) - 3 = 0$

(c) $3(x^2 + 2y^2 z^2) + 2(4yz - 3zx) - 6 = 0$

(d) $xy + yx + zx = xyz$

74. The Equation of the cylinder whose generators are $x = -\frac{y}{2} = \frac{z}{3}$ and whose guiding curve is the ellipse $x^2 + 2y^2 = 1, z = 0$ is

(a) $xy \pm yx \pm zx = 0$

(b) $xx_1 \pm yy_1 \pm zz_1 = r^2$

(c) $xx_1 + yy_1 + zz_1 + r^2 = 0$

(d) $xy \pm yx \pm zx = xyz$

75. Which is the correct form of scalar triple product?

(a) $\vec{a}.\,(\vec{b} \times \vec{c})$

(b) $\vec{a}\,(\vec{b}.\,\vec{c})$

(c) $\vec{a} \times\, (\vec{b} \times \vec{c})$

(d) $\vec{a}.\,(\vec{b}.\,\vec{c})$

76. The geometrical meaning of scalar triple product is

(a) parallel

(b) triangle

(c) parallelepiped

(d) quadrilateral

77. The value of $\vec{a} \times\, (\vec{b} \times \vec{c}) + \vec{b} \times\, (\vec{c} \times \vec{a}) + \vec{c} \times\, (\vec{a} \times \vec{b})$ is

(a) 1

(b) 0

(c) -1

(d) none of the above

78. If $\vec{a'}, \vec{b'}, \vec{c'}$ and $\vec{a}, \vec{b}, \vec{c}$ be reciprocal system, then $[\vec{a'}\ \vec{b'}\ \vec{c'}]\ [\vec{a}\ \vec{b}\ \vec{c}]$

(a) 0

(b) -1

(c) 2

(d) 1

79. The value of Curl (gradϕ) is

(a) 0

(b) -1

(c) 1

(d) none of the above

80. Which is the correct form of div $(\phi\,\vec{a})$?

(a) div. $\vec{a} + \vec{a}.$ gradϕ

(b) div $\vec{a} + \vec{a}.$ gradϕ

(c) div$\times\,\vec{a} + \vec{a}\times$ gradϕ

(d) curl $\vec{a} - \vec{a}.$ curlϕ

81. An equation is said to be a differential equation if it consists of at least a

(a) variable

(b) derivative

(c) coefficient

(d) boundary value

82. An example of first order linear differential equation in standard form is

(a) $y' + xy = 2$
(b) $y'' + y = x$
(c) $y' = y^2 + 2x$
(d) $y' + \frac{x}{y} = 0$

83. The auxiliary equation of the differential equation $ay'' + by' + cy = 0$ is

(a) $ar^3 + br^2 + cr = 0$
(b) $ar^2 - br + c = 0$
(c) $ar^2 + br + c = 0$
(d) $ar^2 + br + c$

84. The integrating factor of the differential equation $\frac{dy}{dx} + p(x)y = q(x)$ is

(a) $e^{\int p'(x)dx}$
(b) $e^{\int p(x)dx}$
(c) $e^{\int -p(x)dx}$
(d) $e^{-\int p(x)dx}$

85. The differential equation $t^2\frac{d^2y}{dt^2} + \alpha t\frac{dy}{dt} + \beta y = 0, t > 0$ is called

(a) Euler equation
(b) harmonic equation
(c) Stock's equation
(d) wave equation

86. The general solution to the second order nonhomogeneous linear equation $y'' + p(x)y' + q(x)y = g(x)$is of the form

(a) $\phi(x) = c_1y_1(x) - c_2y_2(x) + Y(x)$
(b) $\phi(x) = c_1y_1(x) + c_2y_2(x) - Y(x)$
(c) $\phi(x) = -c_1y_1(x) + c_2y_2(x) + Y(x)$
(d) $\phi(x) = c_1y_1(x) + c_2y_2(x) + Y(x)$

87. The characteristic equation of $y'' - 2y' - 3y = 3e^{2t}$ is

(a) $(r - 3)(r + 1) = 0$
(b) $(r + 3)(r + 1) = 0$
(c) $(3 - 2r)(r + 1) = 0$
(d) $(r - 3)(r - 1) = 0$

88. The Heat equation is

(a) $\nabla^2 u = 0$
(b) $\nabla^2 u = \frac{1}{\sigma^2}\frac{\delta u}{\delta t}$
(c) $\nabla^2 u = \frac{1}{a^2}\frac{\delta^2 u}{\delta t^2}$
(d) $\nabla^2 u = \frac{1}{\sigma}\frac{\delta u}{\delta t}$

89. The standard form of linear partial differential equation of order one is where P, Q, R are functions of x

(a) $Pp - Qq = R$
(b) $Pp + Qq^2 = R$
(c) $Pp + Qq = R$
(d) $Pp^2 + Qq = R$

90. The statement "The general solution of linear partial differential equation is $\phi(u, v) = 0$ where ϕ is an arbitrary function and $u(x, y, z) = c_1$ and $v(x, y, z) = c_2$ form a solution of $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$, where P, Q, R are functions of x, y and z " is called

(a) Langrange's theorem
(b) Euler's theorem
(c) Hamilton's theorem
(d) Green's theorem

Attempt either from Mechanics or Linear Programming.

Mechanics

91. The force in a string connecting two particles has a tendency to bring the particles together is called

(a) tension
(b) thrust
(c) buoyancy
(d) reaction

92. The necessary and sufficient conditions for the equilibrium of coplanar and concurrent forces are

(a) the resultant or their resolved parts along two perpendicular directions are zero
(b) the resultant and their resolved parts along two non-perpendicular directions are zero
(c) the resultant but not their resolved parts along two perpendicular directions are zero
(d) the resultant and their resolved parts along two perpendicular directions are zero

93. The magnitude of the resultant force R of the two perpendicular forces P and Q is

(a) $\sqrt{P^2 - Q^2}$
(b) $\sqrt{P + Q}$
(c) $\sqrt{P^2 + Q^2}$
(d) $P^2 + Q^2$

94. The number n of complete oscillations in one second is given by

(a) $\frac{1}{T - 1}$
(b) $\frac{1}{T}$
(c) $\frac{1}{\sqrt{T}}$
(d) $\frac{1}{T + 1}$

95. Let v be the velocity of a particle $P(r, \theta)$ and p be the perpendicular distance from the origin to the tangent at P. The relation between angular and linear velocities is

(a) $(\dot{r\theta}) = \frac{vp}{r^2}$

(b) $\dot{\theta} = \frac{vp}{r^2}$

(c) $\theta = \frac{vp}{r^2}$

(d) $\dot{\theta} = \frac{\dot{v}p}{r^2}$

96. If three coplanar forces, acting in one plane upon a rigid body, keep it in equilibrium, they must

(a) either meet in a point or be parallel

(b) neither meet in a point nor be parallel

(c) either meet in a point or be perpendicular

(d) neither meet in a point nor be perpendicular

97. The relationship between the coefficient of friction μ and the angle of friction λ is

(a) $\mu = \csc \lambda$

(b) $\mu = \tan \lambda$

(c) $\mu = -\tan \lambda$

(d) $\lambda = \tan \mu$

98. The centre of gravity of a uniform triangular area lies at

(a) the point where the medians meet

(b) one vertex

(c) the mid point of a side

(d) the incentre

99. The virtual work done by the tension of an inextensible string is

(a) negative

(b) positive

(c) zero

(d) not fixed

100. If a point moves along a circle, its angular velocity about any point on the circle is ... of that about the centre.

(a) $\frac{2}{3}$

(b) $\frac{1}{3}$

(c) $\frac{3}{2}$

(d) $\frac{1}{2}$

Linear Programming

91 . A necessary condition for a minimization of a real valued function $f(x)$ at a point $x = c$ is that

(a) $f'(c)$ exists and equals to 0

(b) $f'(c)$ exists and yields a positive value

(c) $f'(c)$ exists and yields a negative value

(d) $f'(c)$ does not exist

92 . Consider any LPP $\min\{cx \mid Ax = b, x \geq 0\}$. Then following statement is not valid

(a) the set of extreme points is sufficient for finding a minimum solution

(b) for each extreme point there exists a cost vector such that this point becomes optimal
(c) an optimal solution does not exist at extreme point
(d) there is no guarantee of an existence of an integer solution

93 . Consider any LPP $\min\{c(x) \mid Ax \geq b, x \geq 0, x \in R^n\}$. Then the true statement is

(a) the function $c(x)$ must be nonlinear
(b) the components of feasible x may be negative
(c) the x must be an integer vector
(d) the objective function and the constraints are linear

94 . Suppose that an unconstrained maximization problem P has an optimal value $F(P)$ and the optimal value of the corresponding restricted problem is $F(R)$. It holds for the objective values

(a) $F(P) \leq F(R)$
(b) $F(P) \geq F(R)$
(c) they are always unequal
(d) they are always equal

95 . For a (primal, dual) pair of a linear programming, the false statement is

(a) complementary slackness conditions do not hold
(b) both are linear programming problems
(c) if the primal has a finite optimal solution, then so does the dual
(d) if optimal solutions exist, then the values are equal

96 . In any linear programming problem, a change on the set of constraints may change in the

(a) feasible solutions
(b) optimal value
(c) optimal solutions
(d) any of the above

97 . A linear programming problem is solved by using a

(a) simplex method
(b) steepest descent method
(c) Netwon's method
(d) none of the above methods

98 . The minimum value of the function $f(x) = 2x^3 - 21x^2 + 36x + 5$ occurs at

(a) $x = 1$
(b) $x = 0$
(c) $x = 6$
(d) neither of them

99 . The linear programming problem $\min\{c'x \mid Ax \geq b, x \in R^n_{\geq 0}\}$ is in the form

(a) canonical
(b) general
(c) standard
(d) none of the above

100 . The max-flow and min-cut theorem states that

(a) the maximum flow value never equals to the minimum cut
(b) the maximum flow value equals to the minimum cut for any feasible solution
(c) the maximum flow value equals to the minimum cut when an optimal solution is obtained
(d) none of the above statement is correct